Resistance distances in corona and neighborhood corona graphs with Laplacian generalized inverse approach

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Abstract

Let G_1 and G_2 be two graphs on disjoint sets of n_1 and n_2 vertices, respectively. The corona of graphs G_1 and G_2, denoted by $G_1 \circ G_2$, is the graph formed from one copy of G_1 and n_1 copies of G_2 where the i-th vertex of G_1 is adjacent to every vertex in the i-th copy of G_2. The neighborhood corona of G_1 and G_2, denoted by $G_1 \diamond G_2$, is the graph obtained by taking one copy of G_1 and n_1 copies of G_2 and joining every neighbor of the i-th vertex of G_1 to every vertex in the i-th copy of G_2 by a new edge. In this paper, the Laplacian generalized inverse for the graphs $G_1 \circ G_2$ and $G_1 \diamond G_2$ are investigated, based on which the resistance distances of any two vertices in $G_1 \circ G_2$ and $G_1 \diamond G_2$ can be obtained. Moreover, some examples as applications are presented, which illustrate the correction and efficiency of the proposed method.

Keywords: Laplacian matrix; Generalized inverse; Moore-Penrose inverse; Schur complement; Resistance distance

1 Introduction

All graphs considered in this paper are simple and undirected. Let $G = (V(G), E(G))$ be a graph with vertex set $V(E) = \{v_1, v_2, \dots, v_n\}$ and edge set $E(G) = \{e_1, e_2, \dots, e_m\}$. The adjacency matrix of G, denoted by $A(G)$, is the $n \times n$ matrix whose (i, j)-entry is 1 if v_i and v_j are adjacent in G and 0 otherwise. Denote $D(G)$ to be the diagonal matrix with diagonal entries $d_G(v_1), d_G(v_2), \dots, d_G(v_n)$. The Laplacian matrix of G is defined as $L(G) = D(G) - A(G)$. For other undefined notations and terminology from graph theory, the readers may refer to [1] and the references therein.

The conventional distance between vertices v_i and v_j, denoted by $d_{i,j}$, is the length of a shortest path between them. Klein and Randić [2] introduced a new distance function named resistance distance based on electrical network theory, the resistance distance between vertices i and j, denoted by r_{ij}, is defined to be the effective electrical resistance between them if each edge of G is replaced by a unit resistor [2]. For more information on resistance distance of graphs, the readers are referred to the most recent papers [3, 5, 6, 7, 8, 15, 16, 17, 18].

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Until now, many graph operations such as the Cartesian product, the Kronecker product, the corona and neighborhood corona graphs have been introduced in [10, 11, 12, 9, 13]. Let G_1 and G_2 be two vertex disjoint graphs. The following definition comes from [11].

Definition 1.1 *(see [11]) Let G_1 and G_2 be two graphs on disjoint sets of n_1 and n_2 vertices, respectively. The corona of two graphs G_1 and G_2 is the graph $G = G_1 \circ G_2$ formed from one copy of G_1 and n_1 copies of G_2 where the i-th vertex of G_1 is adjacent to every vertex in the i-th copy of G_2.*

The neighborhood corona, which is a variant of the corona operation, was recently introduced in [13].

Definition 1.2 *(see [13]) Let G_1 and G_2 be two graphs on disjoint sets of n_1 and n_2 vertices, respectively. The neighborhood corona of G_1 and G_2, denoted by $G_1 \diamond G_2$, is the graph obtained by taking one copy of G_1 and n_1 copies of G_2 and joining every neighbor of the i-th vertex of G_1 to every vertex in the i-th copy of G_2 by a new edge.*

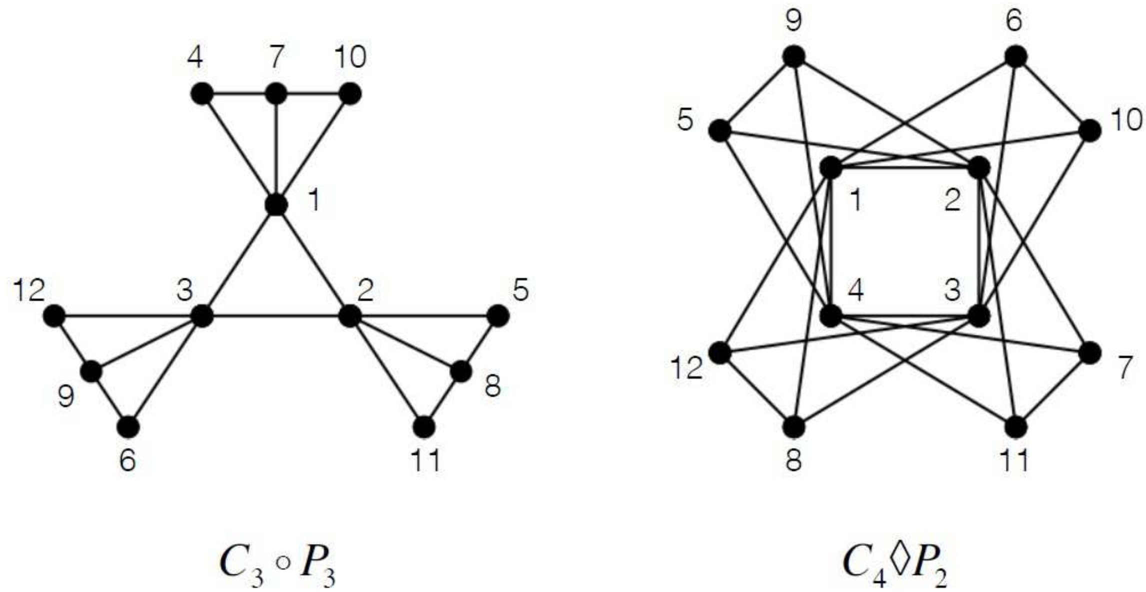

Figure 1: $C_3 \circ P_3$ and $C_4 \diamond P_2$.

Let P_n and C_n denote a path and cycle with n vertices, respectively. From the definitions, Figure 1 shows the graphs $C_3 \circ P_3$ and $C_4 \diamond P_2$.

Bu et al. investigated resistance distances in subdivision-vertex join and subdivision-edge join of graphs [3]. Motivated by the results, in this paper, we further explored the Laplacian generalized

inverse for the corona and neighborhood corona graphs, based on which all the resistance distances between arbitrary two vertices can be directly obtained via simple calculations.

2 Preliminaries and Lemmas

At the beginning of this section, we review some concepts in matrix theory. Let A be a matrix, X is called the $\{1\}$ inverse of A and denoted by $A^{\{1\}}$, if X satisfies the following condition: $AXA = A$. Given a square matrix A, the group inverse of A, denoted by $A^{\#}$, is the unique matrix X that satisfies matrix equations [3] $(I).AXA = A$, $(II).XAX = X$, $(III).AX = XA$. If A is real symmetric, then $A^{\#}$ exists and $A^{\#}$ is a symmetric $\{1\}$-inverse of A. In fact, $A^{\#}$ is equal to the Moore-Penrose inverse of A since A is symmetric [3].

The Kronecker product $A \bigotimes B$ of two matrices $A = (a_{ij})_{m\times n}$ and $B = (b_{ij})_{p\times q}$ is the $mp \times nq$ matrix obtained from A by replacing each element a_{ij} by $a_{ij}B$. The reader is referred to [14] for other properties of the Kronecker product not mentioned here.

It is known that resistance distances in a connected graph G can be obtained from any $\{1\}$-inverse of $L(G)$ according to the following lemma (see [3]).

Lemma 2.1 *(see [3]) Let G be a connected graph, and $(L_G)_{ij}$ denote the (i, j)-entry of L_G. Then*

$$r_{ij}(G) = (L_G^{(1)})_{ii} + (L_G^{(1)})_{jj} - (L_G^{(1)})_{ij} - (L_G^{(1)})_{ji} = (L_G^{\#})_{ii} + (L_G^{\#})_{jj} - 2(L_G^{\#})_{ij}.$$

Lemma 2.2 *(see [3]) Let $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ be a nonsingular matrix. If A and D are nonsingular, then*

$$M^{-1} = \begin{bmatrix} A^{-1} + A^{-1}BS^{-1}CA^{-1} & -A^{-1}BS^{-1} \\ -S^{-1}CA^{-1} & S^{-1} \end{bmatrix},$$

where $S = D - CA^{-1}B$ is the Schur complement of A in M.

The following similar result holds for Laplacian matrix of a connected graph.

Lemma 2.3 *(see [4]) Let $L = \begin{bmatrix} L_1 & L_2 \\ L_2^T & L_3 \end{bmatrix}$ be the Laplacian matrix of a connected graph. If L_1 is nonsingular, then $X = \begin{bmatrix} L_1^{-1} + L_1^{-1}L_2S^{\#}L_2^TL_1^{-1} & -L_1^{-1}L_2S^{\#} \\ -S^{\#}L_2^TL_1^{-1} & S^{\#} \end{bmatrix}$ is a symmetric $\{1\}$-inverse of L, where $S = L_3 - L_2^TL_1^{-1}L_2$.*

3 The Laplacian generalized inverse for graphs $G_1 \circ G_2$ and $G_1 \diamond G_2$

3.1 The Laplacian generalized inverse for graph $G_1 \circ G_2$

Let $\mathbf{1}_n$ and $J_{n\times n}$ be all-one column vector of dimensions n and all-one $n \times n$ matrix, respectively.

Theorem 3.1 Let G_1 be an r_1-regular graph with n_1 vertices and m_1 edges, and G_2 an arbitrary graph with n_2 vertices, then the following matrix

$$\left[\begin{array}{c|c} L_1^{-1}+L_1^{-1}L_2S^{\#}L_2^TL_1^{-1} & -L_1^{-1}L_2S^{\#} \\ \hline -S^{\#}L_2^TL_1^{-1} & S^{\#} \end{array}\right]$$

is a symmetric $\{1\}$-inverse of $L_{(G_1\circ G_2)}$, where

$L_1=[L_{(G_1)}+n_2I_{n_1}], L_2=[-\mathbf{1}_{n_2}^T\otimes I_{n_1}], L_3=(L(G_2)+I_{n_2})\otimes I_{n_1}, S=\Big[L_3-J_{n_2\times n_2}\otimes L_1^{-1}\Big].$

Proof. Let G_1 be an arbitrary r_1-regular graphs with n_1 vertices and m_1 edges, and G_2 an arbitrary graphs with n_2 vertices, respectively. Label the vertices of $G_1\circ G_2$ as follows. Let $V(G_1)=\{v_1,v_2,\ldots,v_{n_1}\}$ and $V(G_2)=\{w_1,w_2,\ldots,w_{n_2}\}$. For $i=1,2,\ldots,n_1$, let $w_1^i,w_2^i,\ldots,w_{n_2}^i$ denote the vertices of the i-th copy of G_2, with the understanding that w_j^i is the copy of w_j for each j. Denote $W_j=\{w_j^1,w_j^2,\ldots,w_j^{n_1}\}$, for $j=1,2,\ldots,n_2$. Then

$$V(G_1)\bigcup[W_1\bigcup W_2\bigcup\ldots\bigcup W_{n_2}] \tag{1}$$

is a partition of $V(G_1\circ G_2)$. Obviously, the degrees of the vertices of $G_1\circ G_2$ are: $d_{G_1\circ G_2}(e_i)=2$, for $i=1,2,\ldots,m_1$, $d_{G_1\circ G_2}(v_i)=n_2+d_{G_1}(v_i)$, for $i=1,2,\ldots,n_1$, and $d_{G_1\circ G_2}(w_j^i)=d_{G_2}(w_j)+1$, for $i=1,2,\ldots,n_1, j=1,2,\ldots,n_2$.

Since G_1 is an r_1-regular graph, we have $D(G_1)=r_1I_{n_1}$. With respect to the partition (1), then the Laplacian matrix of $G_1\circ G_2$ can be written as

$$L(G_1\circ G_2)=\left[\begin{array}{c|c} L(G_1)+n_2I_{n_1} & -\mathbf{1}_{n_2}^T\otimes I_{n_1} \\ \hline -\mathbf{1}_{n_2}\otimes I_{n_1} & (L(G_2)+I_{n_2})\otimes I_{n_1} \end{array}\right].$$

We begin with the calculation S. For convenience, let

$L_1=[L_{(G_1)}+n_2I_{n_1}], L_2=[-\mathbf{1}_{n_2}^T\otimes I_{n_1}]$, $L_2^T=[-\mathbf{1}_{n_2}\otimes I_{n_1}], L_3=(L(G_2)+I_{n_2})\otimes I_{n_1}$.

By Lemma 2.3, we have

$$\begin{aligned} S &= \Big[L(G_2)+I_{n_2}\Big]\otimes I_{n_1}-\big[\ -\mathbf{1}_{n_2}\otimes I_{n_1}\ \big]\big[\ L(G_1)+n_2I_{n_1}\ \big]^{-1}\big[\ -\mathbf{1}_{n_2}^T\otimes I_{n_1}\ \big] \\ &= \Big[L(G_2)+I_{n_2}\Big]\otimes I_{n_1}-J_{n_2\times n_2}\otimes\big[\ L(G_1)+n_2I_{n_1}\ \big]^{-1} \\ &= L_3-J_{n_2\times n_2}\otimes L_1^{-1}. \end{aligned}$$

Based on Lemma 2.3, the following matrix

$$\left[\begin{array}{c|c} L_1^{-1}+L_1^{-1}L_2S^{\#}L_2^TL_1^{-1} & -L_1^{-1}L_2S^{\#} \\ \hline -S^{\#}L_2^TL_1^{-1} & S^{\#} \end{array}\right]$$

is a symmetric $\{1\}$-inverse of $L_{(G_1\circ G_2)}$, where

$L_1=[L_{(G_1)}+n_2I_{n_1}], L_2=[-\mathbf{1}_{n_2}^T\otimes I_{n_1}], L_3=(L(G_2)+I_{n_2})\otimes I_{n_1}, S=\Big[L_3-J_{n_2\times n_2}\otimes L_1^{-1}\Big].$ ■

3.2 The Laplacian generalized inverse for graph $G_1 \diamond G_2$

When G_1 is a regular graph, we obtain the Laplacian generalized inverse for graph $G_1 \diamond G_2$ as follows.

Theorem 3.2 Let G_1 be an r_1-regular graph with n_1 vertices and m_1 edges, and G_2 an arbitrary graph with n_2 vertices, then the following matrix

$$\left[\begin{array}{c|c} L_1^{-1} + L_1^{-1} L_2 S^{\#} L_2^T L_1^{-1} & -L_1^{-1} L_2 S^{\#} \\ \hline -S^{\#} L_2^T L_1^{-1} & S^{\#} \end{array}\right]$$

is a symmetric $\{1\}$-inverse of $L_{(G_1 \diamond G_2)}$, where $L_1 = L(G_1) + n_2 D(G_1), L_2 = -\mathbf{1}_{n_2}^T \otimes A(G_1)$, $L_3 = L(G_2) \otimes I_{n_1} + I_{n_2} \otimes D(G_1), S = L_3 - J_{n_2 \times n_2} \otimes [A(G_1)^T L_1^{-1} A(G_1)]$.

Proof. We label the vertices of $G_1 \diamond G_2$ as follows. Let $V(G_1) = \{v_1, v_2, \ldots, v_{n_1}\}$, and $V(G_2) = \{w_1, w_2, \ldots, w_{n_2}\}$. For $i = 1, 2, \ldots, n_1$, let $w_1^i, w_2^i, \ldots, w_{n_2}^i$ denote the vertices of the i-th copy of G_2, with the understanding that w_j^i is the copy of w_j for each j. Denote $U_j = \{w_j^1, w_j^2, \ldots, w_j^{n_1}\}$, for $j = 1, 2, \ldots, n_2$. Then

$$V(G_1) \bigcup \left[U_1 \bigcup U_2 \bigcup \cdots \bigcup U_{n_2}\right] \tag{2}$$

is a partition of $V(G_1 \diamond G_2)$. Clearly, the degrees of the vertices of $G_1 \diamond G_2$ are:

$d_{G_1 \diamond G_2}(v_i) = (n_2 + 1) d_{G_1}(v_i)$, for $i = 1, 2, \ldots, n_1$, and $d_{G_1 \diamond G_2}(w_j^i) = d_{G_2}(w_j) + d_{G_1}(v_i)$, for $i = 1, 2, \ldots, m_1, j = 1, 2, \ldots, n_2$.

Since G_1 is an r_1-regular graph, we have $D(G_1) = r_1 I_{n_1}$. With respect to the partition (2), then the Laplacian matrix of $G_1 \diamond G_2$ can be written as

$$L(G_1 \diamond G_2) = \left[\begin{array}{c|c} L(G_1) + n_2 D(G_1) & -\mathbf{1}_{n_2}^T \otimes A(G_1) \\ \hline -\mathbf{1}_{n_2} \otimes A(G_1)^T & L(G_2) \otimes I_{n_1} + I_{n_2} \otimes D(G_1) \end{array}\right].$$

For convenience, let $L_1 = L(G_1) + n_2 D(G_1), L_2 = -\mathbf{1}_{n_2}^T \otimes A(G_1), L_2^T = -\mathbf{1}_{n_2} \otimes A(G_1)^T$, $L_3 = L(G_2) \otimes I_{n_1} + I_{n_2} \otimes D(G_1)$.

Similarly, by Lemma 2.3, we have

$$\begin{aligned} S &= [L(G_2) \otimes I_{n_1} + I_{n_2} \otimes D(G_1)] - \left[-\mathbf{1}_{n_2} \otimes A(G_1)^T\right] \left[\ L(G_1) + n_2 D(G_1)\ \right]^{-1} \left[\ -\mathbf{1}_{n_2}^T \otimes A(G_1)\ \right] \\ &= L_3 - J_{n_2 \times n_2} \otimes [A(G_1)^T L_1^{-1} A(G_1)]. \end{aligned}$$

Based on Lemma 2.3, the following matrix

$$\left[\begin{array}{c|c} L_1^{-1} + L_1^{-1} L_2 S^{\#} L_2^T L_1^{-1} & -L_1^{-1} L_2 S^{\#} \\ \hline -S^{\#} L_2^T L_1^{-1} & S^{\#} \end{array}\right]$$

is a symmetric $\{1\}$-inverse of $L_{(G_1 \circ G_2)}$, where $L_1 = L(G_1) + n_2 D(G_1), L_2 = -\mathbf{1}_{n_2}^T \otimes A(G_1)$, $L_3 = L(G_2) \otimes I_{n_1} + I_{n_2} \otimes D(G_1), S = L_3 - J_{n_2 \times n_2} \otimes [A(G_1)^T L_1^{-1} A(G_1)]$. ■

4 Applications and some examples

As an application of the proposed theorems, we present some examples to show all the resistance distances of any two vertices in graphs $G_1 \circ G_2$ and $G_1 \diamond G_2$ can be obtained by the proposed method.

Example 4.1 *Laplacian generalized inverse for $C_3 \circ P_3$ and resistance distances matrix.*

The Laplacian matrix $L_{(C_3 \circ P_3)} = \left[\begin{array}{c|c} L(C_3) + 3I_3 & -\mathbf{1}_3^T \otimes I_3 \\ \hline -\mathbf{1}_3 \otimes I_3 & (L(P_3) + I_{n_2}) \otimes I_3 \end{array}\right]$.

Based on Theorem 3.1, we can obtain that

$$L^{\{1\}}_{(C_3 \circ P_3)} = \begin{bmatrix}
\frac{1}{3} & 0 & 0 & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} \\
0 & \frac{1}{3} & 0 & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{9} \\
0 & 0 & \frac{1}{3} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} \\
\frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{53}{72} & \frac{-2}{9} & \frac{-2}{9} & \frac{13}{36} & \frac{-2}{9} & \frac{-2}{9} & \frac{17}{72} & \frac{-2}{9} & \frac{-2}{9} \\
\frac{-1}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-2}{9} & \frac{53}{72} & \frac{-2}{9} & \frac{-2}{9} & \frac{13}{36} & \frac{-2}{9} & \frac{-2}{9} & \frac{17}{72} & \frac{-2}{9} \\
\frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{53}{72} & \frac{-2}{9} & \frac{-2}{9} & \frac{13}{36} & \frac{-2}{9} & \frac{-2}{9} & \frac{17}{72} \\
\frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{13}{36} & \frac{-2}{9} & \frac{-2}{9} & \frac{11}{18} & \frac{-2}{9} & \frac{-2}{9} & \frac{13}{36} & \frac{-2}{9} & \frac{-2}{9} \\
\frac{-1}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-2}{9} & \frac{13}{36} & \frac{-2}{9} & \frac{-2}{9} & \frac{11}{18} & \frac{-2}{9} & \frac{-2}{9} & \frac{13}{36} & \frac{-2}{9} \\
\frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{13}{36} & \frac{-2}{9} & \frac{-2}{9} & \frac{11}{18} & \frac{-2}{9} & \frac{-2}{9} & \frac{13}{36} \\
\frac{2}{9} & \frac{-1}{9} & \frac{-1}{9} & \frac{17}{72} & \frac{-2}{9} & \frac{-2}{9} & \frac{13}{36} & \frac{-2}{9} & \frac{-2}{9} & \frac{53}{72} & \frac{-2}{9} & \frac{-2}{9} \\
\frac{-1}{9} & \frac{2}{9} & \frac{-1}{9} & \frac{-2}{9} & \frac{17}{72} & \frac{-2}{9} & \frac{-2}{9} & \frac{13}{36} & \frac{-2}{9} & \frac{-2}{9} & \frac{53}{72} & \frac{-2}{9} \\
\frac{-1}{9} & \frac{-1}{9} & \frac{2}{9} & \frac{-2}{9} & \frac{-2}{9} & \frac{17}{72} & \frac{-2}{9} & \frac{-2}{9} & \frac{13}{36} & \frac{-2}{9} & \frac{-2}{9} & \frac{53}{72}
\end{bmatrix}.$$

By Lemma 2.1 and $L^{\{1\}}_{(C_3 \circ P_3)}$, the resistance distances matrix of $C_3 \circ P_3$ is

$$R_{(C_3 \circ P_3)} = \begin{bmatrix}
0 & \frac{2}{3} & \frac{2}{3} & \frac{5}{8} & \frac{31}{24} & \frac{31}{24} & \frac{1}{2} & \frac{7}{6} & \frac{7}{6} & \frac{5}{8} & \frac{31}{24} & \frac{31}{24} \\
\frac{2}{3} & 0 & \frac{2}{3} & \frac{31}{24} & \frac{5}{8} & \frac{31}{24} & \frac{7}{6} & \frac{1}{2} & \frac{7}{6} & \frac{31}{24} & \frac{5}{8} & \frac{31}{24} \\
\frac{2}{3} & \frac{2}{3} & 0 & \frac{31}{24} & \frac{31}{24} & \frac{5}{8} & \frac{7}{6} & \frac{7}{6} & \frac{1}{2} & \frac{31}{24} & \frac{31}{24} & \frac{5}{8} \\
\frac{5}{8} & \frac{31}{24} & \frac{31}{24} & 0 & \frac{23}{12} & \frac{23}{12} & \frac{5}{8} & \frac{43}{24} & \frac{43}{24} & 1 & \frac{23}{12} & \frac{23}{12} \\
\frac{31}{24} & \frac{5}{8} & \frac{31}{24} & \frac{23}{12} & 0 & \frac{23}{12} & \frac{43}{24} & \frac{5}{8} & \frac{43}{24} & \frac{23}{12} & 1 & \frac{23}{12} \\
\frac{31}{24} & \frac{31}{24} & \frac{5}{8} & \frac{23}{12} & \frac{23}{12} & 0 & \frac{43}{24} & \frac{43}{24} & \frac{5}{8} & \frac{23}{12} & \frac{23}{12} & 1 \\
\frac{1}{2} & \frac{7}{6} & \frac{7}{6} & \frac{5}{8} & \frac{43}{24} & \frac{43}{24} & 0 & \frac{5}{3} & \frac{5}{3} & \frac{5}{8} & \frac{43}{24} & \frac{43}{24} \\
\frac{7}{6} & \frac{1}{2} & \frac{7}{6} & \frac{43}{24} & \frac{5}{8} & \frac{43}{24} & \frac{5}{3} & 0 & \frac{5}{3} & \frac{43}{24} & \frac{5}{8} & \frac{43}{24} \\
\frac{7}{6} & \frac{7}{6} & \frac{1}{2} & \frac{43}{24} & \frac{43}{24} & \frac{5}{8} & \frac{5}{3} & \frac{5}{3} & 0 & \frac{43}{24} & \frac{43}{24} & \frac{5}{8} \\
\frac{5}{8} & \frac{31}{24} & \frac{31}{24} & 1 & \frac{23}{12} & \frac{23}{12} & \frac{5}{8} & \frac{43}{24} & \frac{43}{24} & 0 & \frac{23}{12} & \frac{23}{12} \\
\frac{31}{24} & \frac{5}{8} & \frac{31}{24} & \frac{23}{12} & 1 & \frac{23}{12} & \frac{43}{24} & \frac{5}{8} & \frac{43}{24} & \frac{23}{12} & 0 & \frac{23}{12} \\
\frac{31}{24} & \frac{31}{24} & \frac{5}{8} & \frac{23}{12} & \frac{23}{12} & 1 & \frac{43}{24} & \frac{43}{24} & \frac{5}{8} & \frac{23}{12} & \frac{23}{12} & 0
\end{bmatrix},$$

where r_{ij} denotes resistance distance of two vertices between i and j. ∎

Example 4.2 *Laplacian generalized inverse for $C_4 \diamond P_2$ and resistance distances matrix.*

Completely similar deduction by Theorem 3.2, we can obtain

$$L^{\{1\}}_{(C_4 \diamond P_2)} = \begin{bmatrix}
\frac{5}{24} & 0 & \frac{1}{24} & 0 & \frac{-1}{16} & \frac{1}{16} & \frac{-1}{16} & \frac{1}{16} & \frac{-1}{16} & \frac{1}{16} & \frac{-1}{16} & \frac{1}{16} \\
0 & \frac{5}{24} & 0 & \frac{1}{24} & \frac{1}{16} & \frac{-1}{16} & \frac{1}{16} & \frac{-1}{16} & \frac{1}{16} & \frac{-1}{16} & \frac{1}{16} & \frac{-1}{16} \\
\frac{1}{24} & 0 & \frac{5}{24} & 0 & \frac{-1}{16} & \frac{1}{16} & \frac{-1}{16} & \frac{1}{16} & \frac{-1}{16} & \frac{1}{16} & \frac{-1}{16} & \frac{1}{16} \\
0 & \frac{1}{24} & 0 & \frac{5}{24} & \frac{1}{16} & \frac{-1}{16} & \frac{1}{16} & \frac{-1}{16} & \frac{1}{16} & \frac{-1}{16} & \frac{1}{16} & \frac{-1}{16} \\
\frac{-1}{16} & \frac{1}{16} & \frac{-1}{16} & \frac{1}{16} & \frac{3}{8} & \frac{-1}{8} & 0 & \frac{-1}{8} & \frac{1}{8} & \frac{-1}{8} & 0 & \frac{-1}{8} \\
\frac{1}{16} & \frac{-1}{16} & \frac{1}{16} & \frac{-1}{16} & \frac{-1}{8} & \frac{3}{8} & \frac{-1}{8} & 0 & \frac{-1}{8} & \frac{1}{8} & \frac{-1}{8} & 0 \\
\frac{-1}{16} & \frac{1}{16} & \frac{-1}{16} & \frac{1}{16} & 0 & \frac{-1}{8} & \frac{3}{8} & \frac{-1}{8} & 0 & \frac{-1}{8} & \frac{1}{8} & \frac{-1}{8} \\
\frac{1}{16} & \frac{-1}{16} & \frac{1}{16} & \frac{-1}{16} & \frac{-1}{8} & 0 & \frac{-1}{8} & \frac{3}{8} & \frac{-1}{8} & 0 & \frac{-1}{8} & \frac{1}{8} \\
\frac{-1}{16} & \frac{1}{16} & \frac{-1}{16} & \frac{1}{16} & \frac{1}{8} & \frac{-1}{8} & 0 & \frac{-1}{8} & \frac{3}{8} & \frac{-1}{8} & 0 & \frac{-1}{8} \\
\frac{1}{16} & \frac{-1}{16} & \frac{1}{16} & \frac{-1}{16} & \frac{-1}{8} & \frac{1}{8} & \frac{-1}{8} & 0 & \frac{-1}{8} & \frac{3}{8} & \frac{-1}{8} & 0 \\
\frac{-1}{16} & \frac{1}{16} & \frac{-1}{16} & \frac{1}{16} & 0 & \frac{-1}{8} & \frac{1}{8} & \frac{-1}{8} & 0 & \frac{-1}{8} & \frac{3}{8} & \frac{-1}{8} \\
\frac{1}{16} & \frac{-1}{16} & \frac{1}{16} & \frac{-1}{16} & \frac{-1}{8} & 0 & \frac{-1}{8} & \frac{1}{8} & \frac{-1}{8} & 0 & \frac{-1}{8} & \frac{3}{8}
\end{bmatrix}.$$

By Lemma 2.1 and $L^{\{1\}}_{(C_4 \diamond P_2)}$, the resistance distances matrix of $C_4 \diamond P_2$ is

$$R_{(C_4 \diamond P_2)} = \begin{bmatrix}
0 & \frac{5}{12} & \frac{1}{3} & \frac{5}{12} & \frac{17}{24} & \frac{11}{24} & \frac{17}{24} & \frac{11}{24} & \frac{17}{24} & \frac{11}{24} & \frac{17}{24} & \frac{11}{24} \\
\frac{5}{12} & 0 & \frac{5}{12} & \frac{1}{3} & \frac{11}{24} & \frac{11}{24} & \frac{17}{24} & \frac{11}{24} & \frac{17}{24} & \frac{17}{24} & \frac{11}{24} & \frac{17}{24} \\
\frac{1}{3} & \frac{5}{12} & 0 & \frac{5}{12} & \frac{17}{24} & \frac{11}{24} & \frac{17}{24} & \frac{11}{24} & \frac{17}{24} & \frac{11}{24} & \frac{17}{24} & \frac{11}{24} \\
\frac{5}{12} & \frac{1}{3} & \frac{5}{12} & 0 & \frac{11}{24} & \frac{17}{24} & \frac{11}{24} & \frac{17}{24} & \frac{11}{24} & \frac{17}{24} & \frac{11}{24} & \frac{17}{24} \\
\frac{17}{24} & \frac{11}{24} & \frac{17}{24} & \frac{11}{24} & 0 & 1 & \frac{3}{4} & 1 & \frac{1}{2} & 1 & \frac{3}{4} & 1 \\
\frac{11}{24} & \frac{17}{24} & \frac{11}{24} & \frac{17}{24} & 1 & 0 & 1 & \frac{3}{4} & 1 & \frac{1}{2} & 1 & \frac{3}{4} \\
\frac{17}{24} & \frac{11}{24} & \frac{17}{24} & \frac{11}{24} & \frac{3}{4} & 1 & 0 & 1 & \frac{3}{4} & 1 & \frac{1}{2} & 1 \\
\frac{11}{24} & \frac{17}{24} & \frac{11}{24} & \frac{17}{24} & 1 & \frac{3}{4} & 1 & 0 & 1 & \frac{3}{4} & 1 & \frac{1}{2} \\
\frac{17}{24} & \frac{11}{24} & \frac{17}{24} & \frac{11}{24} & \frac{1}{2} & 1 & \frac{3}{4} & 1 & 0 & 1 & \frac{3}{4} & 1 \\
\frac{11}{24} & \frac{17}{24} & \frac{11}{24} & \frac{17}{24} & 1 & \frac{1}{2} & 1 & \frac{3}{4} & 1 & 0 & 1 & \frac{3}{4} \\
\frac{17}{24} & \frac{11}{24} & \frac{17}{24} & \frac{11}{24} & \frac{3}{4} & 1 & \frac{1}{2} & 1 & \frac{3}{4} & 1 & 0 & 1 \\
\frac{11}{24} & \frac{17}{24} & \frac{11}{24} & \frac{17}{24} & 1 & \frac{3}{4} & 1 & \frac{1}{2} & 1 & \frac{3}{4} & 1 & 0
\end{bmatrix},$$

where r_{ij} denotes resistance distance of two vertices between i and j. ■

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